CALCULATING HIGHER-ORDER DETERMINANTS USING PROGRAMS

Raximova Feruza Saidovna,
Payziyeva Munira Tairovna,
Atayeva Asal Adilbekovna,
Tashkent University of Information Technologies
Tursunaliyev Ozodbek Bahrom o`g`li,
Iskandarova Dilafruz Sharofaddin qizi,
Tashkent University of Information Technologies (students)

Annotation

It can be said that the problem of calculating matrix determinants forms the foundation of linear algebra. This article presents methods and software for calculating higher-order determinants.

Keywords: Matrix, determinant, Laplace method, minor, algebraic complement.

Аннотация

Вычисления определителя матриц высших поряков является важной темой линейной алгебры. В данной статье представлены методы и коды программ для вычисления определителей высших порядков.

Ключевые слова: Матрица, определитель, метод Лапласа, минор, алгебраическое дополнение.

Guiding students of information technology fields in higher educational institutions to use software and programs when solving problems related to mathematical subjects facilitates the learning of the covered topics and increases students' interest in the lessons. Taking this into account, this article presents methods for calculating determinant values using minors and algebraic complements, and the results of examples are compared with those obtained through software. As we know, a determinant is a scalar quantity that, after expressing a multidimensional Euclidean space in the form of a square matrix, represents the measure that determines its "stretching" or "compression" in a certain direction.

A single element of a matrix is a first-order determinant, and the value of a first-order determinant is equal to that number itself. A second-order determinant $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$ is defined as the number determined by the following equality. There are several methods for calculating third-order determinants, such as the triangular

(Sarrus) method, and for higher-order determinants — methods such as reducing the order, transforming one row into zeros, or converting the matrix into an upper (or lower) triangular form for calculation.

Another method for calculating higher-order determinants is to find the sum of the products of all possible k-order minors of the selected k rows (or columns) of an n-order determinant and their corresponding algebraic complements.

$$detA = \sum_{\substack{i_1<i_2<\cdots<i_k \\ j_1<j_2<\cdots<j_k}} M_{j_1,j_2,\ldots,j_k}^{i_1,i_2,\ldots,i_k} \cdot A_{j_1,j_2,\ldots,j_k}^{i_1,i_2,\ldots,i_k}. \qquad (*)$$

As an example, let us calculate the following determinant using the method of minors and algebraic complements:

$$\Delta = \begin{vmatrix} 2 & -1 & 3 & 4 \\ 1 & 2 & -2 & 5 \\ 3 & 4 & 1 & -1 \\ 2 & 3 & 5 & 2 \end{vmatrix}.$$

Solution. (*) We use the formula, and adapting this formula to our example, we will express it in terms of second-order minors:

$$\Delta=$$

$$= M_{1,2}^{1,2} \cdot A_{1,2}^{1,2} + M_{1,3}^{1,2} \cdot A_{1,3}^{1,2} + M_{1,4}^{1,2} \cdot A_{1,4}^{1,2} + M_{2,3}^{1,2} \cdot A_{2,3}^{1,2} + M_{2,4}^{1,2} \cdot A_{2,4}^{1,2} + M_{3,4}^{1,2} \cdot A_{3,4}^{1,2} =$$

$$= \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} \cdot (-1)^{1+2+1+2} \cdot \begin{vmatrix} 1 & -1 \\ 5 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} \cdot (-1)^{1+2+1+3} \cdot \begin{vmatrix} 4 & -1 \\ 3 & 2 \end{vmatrix} +$$

$$+ \begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix} \cdot (-1)^{1+2+1+4} \cdot \begin{vmatrix} 4 & 1 \\ 3 & 5 \end{vmatrix} + \begin{vmatrix} -1 & 3 \\ 2 & -2 \end{vmatrix} \cdot (-1)^{1+2+2+3} \cdot \begin{vmatrix} 3 & -1 \\ 2 & 2 \end{vmatrix} +$$

$$+ \begin{vmatrix} -1 & 4 \\ 2 & 5 \end{vmatrix} \cdot (-1)^{1+2+2+4} \cdot \begin{vmatrix} 3 & 1 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 4 \\ -2 & 5 \end{vmatrix} \cdot (-1)^{1+2+3+4} \cdot \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} =$$

$$= 5 \cdot (7) - 7 \cdot (-11) + 6 \cdot 17 - 4 \cdot 8 - 13 \cdot (-13) + 23 \cdot (1) = 374.$$

A program was created to solve this example using the above method. We will compare its result with the result of the given example and see that they are the same:

```
def minor(matrix, i, j):
    """Return the minor matrix obtained by removing the i-th row and j-th column"""
    return [row[:j] + row[j+1:] for row in (matrix[:i] + matrix[i+1:])]


def determinant(matrix):
```

```
    """Compute determinant recursively using Laplace expansion"""
    n = len(matrix)

    # Base case: 1x1 matrix
    if n == 1:
        return matrix[0][0]

    # Base case: 2x2 matrix
    if n == 2:
        return matrix[0][0]*matrix[1][1] - matrix[0][1]*matrix[1][0]

    det = 0
    for j in range(n):
        # Apply cofactor expansion along the first row:
        # (-1)^(0+j) * a_0j * det(minor)
        sign = (-1) ** j
        sub = determinant(minor(matrix, 0, j))
        det += sign * matrix[0][j] * sub
    return det

# Example 4x4 matrix
A4 = [
    [2, -1, 3, 4],
    [1, 2, -2, 5],
    [3, 4, 1, -1],
    [2, 3, 5, 2]
]

# Print the determinant
print("Determinant:", determinant(A4))
```

Result:

```
# Example 4x4 matrix
A4 = [
    [2, -1, 3, 4],
    [1, 2, -2, 5],
    [3, 4, 1, -1],
    [2, 3, 5, 2]
]

# Print the determinant
print("Determinant:", determinant(A4))
```

```
PROBLEMS   OUTPUT   DEBUG CONSOLE   TERMINAL   PORTS

PS C:\Users\User\Documents\telebot> python -u "c:\Users\User\Documents\telebot\minoreng.py"
Determinant: 374
```

We will calculate the determinant given above by reducing its order using Laplace's theorem: $\Delta = \begin{vmatrix} 2 & -1 & 3 & 4 \\ 1 & 2 & -2 & 5 \\ 3 & 4 & 1 & -1 \\ 2 & 3 & 5 & 2 \end{vmatrix}$.

Solution. $\Delta = \begin{vmatrix} 2 & -1 & 3 & 4 \\ 1 & 2 & -2 & 5 \\ 3 & 4 & 1 & -1 \\ 2 & 3 & 5 & 2 \end{vmatrix} = a_{41}A_{41} + a_{42}A_{42} + a_{43}A_{43} + a_{44}A_{44} =$

$$= 2 \cdot \begin{vmatrix} 2 & -2 & 5 \\ 4 & 1 & -1 \\ 3 & 5 & 2 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & -2 & 5 \\ 3 & 1 & -1 \\ 2 & 5 & 2 \end{vmatrix} + 3 \cdot \begin{vmatrix} 1 & 2 & 5 \\ 3 & 4 & -1 \\ 2 & 3 & 2 \end{vmatrix} - 4 \cdot \begin{vmatrix} 1 & 2 & -2 \\ 3 & 4 & 1 \\ 2 & 3 & 5 \end{vmatrix} =$$

$$= 2 \cdot (4 + 100 + 6 - 15 + 10 + 16) + 1 \cdot (2 + 75 + 4 - 10 + 12 + 5) + 3 \cdot (8 + 45 - 4 - 40 + 3 - 12) - 4 \cdot (20 - 18 + 4 + 16 - 3 - 30) = 242 + 88 + 44 = 374.$$

By comparing the results from the given example, we can see that they are identical. It is also clear that the same result is obtained when solving this example using the program.

```
def minor(matrix, i, j):
    """Return the minor matrix by removing the i-th row and j-th column"""
    return [row[:j] + row[j+1:] for idx, row in enumerate(matrix) if idx != i]


def minor(matrix, i, j):
    """Return the minor matrix by removing the i-th row and j-th column"""
    return [row[:j] + row[j+1:] for idx, row in enumerate(matrix) if idx != i]


def determinant(matrix):
    """General recursive determinant function"""
```

```
    n = len(matrix)
    if n == 1:
        return matrix[0][0]
    if n == 2:
        return matrix[0][0]*matrix[1][1] - matrix[0][1]*matrix[1][0]

    det = 0
    for j in range(n):
        sign = (-1) ** j
        det += sign * matrix[0][j] * determinant(minor(matrix, 0, j))
    return det

def laplace_by_row(matrix, row):
    """Laplace expansion by a specific row"""
    n = len(matrix)
    det = 0
    print(f"Expansion along row {row+1}:")
    for j in range(n):
        a = matrix[row][j]
        sign = (-1) ** (row + j)
        M = minor(matrix, row, j)
        Mdet = determinant(M)
        term = sign * a * Mdet
        print(f"a[{row+1},{j+1}] = {a},  Minor determinant A[{row+1},{j+1}] = {Mdet},  term
= {term}")
        det += term
    print("Result =", det)
    return det

# Example: 4x4 matrix
A = [
    [2, -1, 3, 4],
    [1, 2, -2, 5],
```

```
    [3, 4, 1, -1],
    [2, 3, 5, 2]
]

# Choose row (1-based index)
row = int(input("Enter the row number (1-based): ")) - 1

laplace_by_row(A, row)
```

Result:

```
Enter the row number (1-based): 1
Expansion along row 1:
a[1,1] = 2,  Minor determinant A[1,1] = 121,  term = 242
a[1,2] = -1,  Minor determinant A[1,2] = 88,  term = 88
a[1,3] = 3,  Minor determinant A[1,3] = 0,  term = 0
a[1,4] = 4,  Minor determinant A[1,4] = -11,  term = 44
Result = 374
```

The advantage of the program's structure is that it allows finding the values of determinants with different elements. The program can also be modified to correctly calculate determinants of various orders.

Several years of experience in teaching higher mathematics to engineering students have shown that guiding students to use programming in learning the subject significantly increases their interest in studying the material.

It also significantly enhances the effectiveness of the lesson.

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