DETERMINING FRACTAL DEMINSION USING THE MASS-RADIUS METHOD

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Abstract. This Mass-Radius (MR) method in the article using eye retinal blood veins network fractal measure of determination mathematician basics and practical application opportunities studied. Retinal vascular of the system geometric structure human in the body microcirculator system status reflection provider complicated is a biological model. This network fractal features analysis to do through many ophthalmological and systematic of diseases early stages determination possible. Mass-Radius method from the center to the periphery directed branching density to evaluate based on the retinal structures to himself the degree of self-similarity logarithmic mass-radius relationship through quantitative represents. Fractal size (D_m) of the image central from the point by radius from pixels number growth legality mathematician in a way describes . This retinal blood method veins network complexity in evaluation high accuracy , stability and to sensitivity has diseases diagnostics for promising approach as This is considered. approach in ophthalmology digital the images analysis to do process automation, human factor reduce and retinal diseases early stages determination opportunities to expand service does.

Key words: fractal measurement, Mass-Radius method, retinal blood veins, fractal analysis, diabetic retinopathy, medical images analysis.

Modern retinal blood in ophthalmology veins network morphological structure study not only eye diseases, but whole organism microcirculator system functional status also important in evaluation diagnostic direction Retinal vascular system human in the body only straight away observable microvessel network is, its changes diabetic retinopathy, hypertensive retinopathy, glaucoma, atherosclerosis, cardiovascular vein diseases and neurodegenerative processes such as different systematic pathologies initial signs reflection This causes retinal blood veins structure quantitative assessment medical the images analysis to do in the field the most current from directions is one.

Traditional ophthalmoscopic assessment methods mainly subjective to the character has is, the results accuracy doctor to experience It depends. This is clinical situations standardized in a way to compare makes it difficult. The last in years mathematician geometry and computer technologies development resulting in retinal images fractal analysis based on analysis to do methods intense is developing. Fractal geometry complicated, self-contained of self- similar structures form and density to describe opportunity gives. Such of structures main parameter - fractal Fractal Dimension (FD) — the size of an object the gap how much complete possession level reflection provider quantitative is an indicator [1].

Retinal blood veins by nature fractal to the feature has is, their branching level and structure complexity of the disease existence and weight level reflection Fractal of the scale decrease microvascular network thinning of capillaries loss and without perfusion zones expansion with related will be. Therefore fractal measurement many researchers as a biomarker by offer is being done [2].

Fractal measure in calculation different methods there is although, their Mass-Radius (MR) method in retinal blood veins anatomical to the structure the most suitable incoming and stable result giver approach is considered. MR method from the center to the periphery grower radii according to blood veins pixels mass analysis to do through structure fractal features This approach evaluates retinal vascular of the system central from the field (view) nerve disk starting branching order into account takes, as a result it is biological in terms of based mathematician the model creates.

Mass-Radius method of the image central in the field geometric to changes high sensitive is a retinal pathology early stages determination opportunity This method gives again one advantage — that image orientation or to the cycle relatively is relatively invariant (stable). Therefore, the MR method medical the images automatic analysis to do, clinical diagnostics and artificial intellect based on medical in systems application for comfortable is [3].

This of the research relevance is that Uzbekistan retinal fractal in the population analysis direction just now is being formed [4]. Local clinical to the information relied on MR method without based on fractal measurements assessment, diagnostic criteria working exit and medical to practice implementation to be — national ophthalmological research for new scientific direction opens.

Mass-Radius (MR) method. Mass-Radius method of the object mass (pixels) number) of it from the center starting measurable to the radius how dependence analysis does. Fractal structures for mass and between the radius The relationship is also level to the law is subject to:

$$M(r) \propto r^{D_M} \tag{1}$$

Here M(r) – with radius r circle inside pixels number (mass), and D_M is the Mass-Radius fractal This equation is logarithmically, the following we get:

$$\log(M(r)) \propto D_M \log(r) \tag{2}$$

So so, the value of D_M log(M(r)) of log(r) to dependency graph slope angle as is found [5].

Formula 2. Mass-Radius method for fractal measure calculation formula:

$$D_M = \frac{d(\log M(r))}{d(\log r)} \tag{3}$$

The MR algorithm stages:

1. Skeletonization made binary image is prepared.
2. Massani calculation for central point (c) is selected. Retinal images for usually vision nerve disk center or of the image geometric center is selected.
3. For radius (r) one how many values sequence is selected (for example, r= 5,10,15 ,..., R_max).
4. Every an r value for center c and with radius r circle inside vein pixels The number (M(r)) is counted.
5. (log(r),log(M(r))) pairs for dotted graphic is drawn.
6. The most small squares method using this to the points the most suitable incoming regression line is held.
7. This of the line slope angle D_M as is taken.

This of the method advantage is that the object branching from the center starting growth dynamics better reflection will bring. However, the result central the point to choose related to be possible [6].

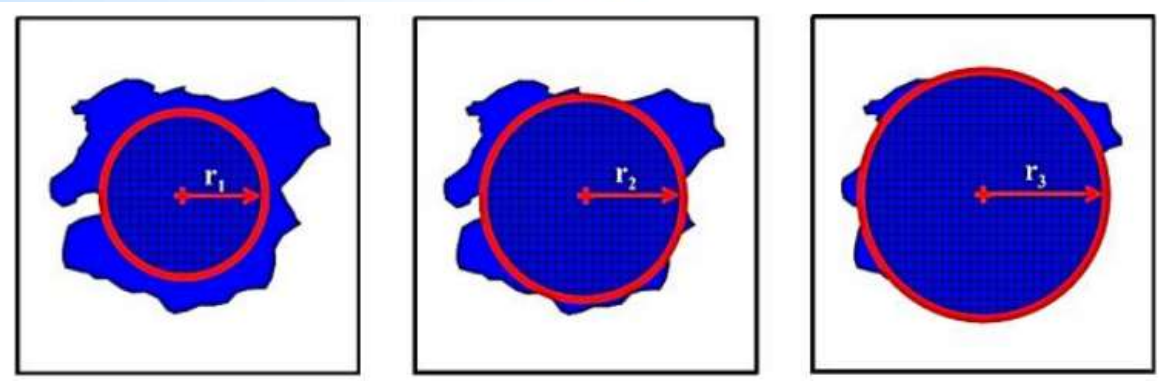

Figure 1. Mass-radius method object in the image expression.

Mass-Radius (MR) method retinal blood veins network fractal features quantitative in evaluation reliable and effective mathematician approach This is of the method the essence of the image central from the point starting radius increasing going circles inside blood vein pixels of mass growth legality logarithmic analysis to do based on [7]. MR method fractal measure in determining from the center to the periphery directed branching density complete into account takes, this and retinal anatomy suitable, biological based the results provides.

This method retinal vascular of the system central (macular) and optic disc areas microstructural changes in determining high to sensitivity has. MR method fractal to measure based mathematician retinal blood model veins complexity reduction, microvessels thinning and without perfusion zones appearance to be such as pathological processes clear reflection In this respect, it is diabetic. retinopathy, hypertensive retinopathy and other microvascular diseases early in stages determination for promising tool is a fractal. MR measurement method using assessment medical the images automated in a way analysis to do, human factor reduce and diagnostic the process standardization opportunity This gives approach clinical in practice new diagnostic biomarkers current of diseases dynamics observation and treatment efficiency objective assessment for wide opportunities creates [8].

So Thus, the Mass-Radius method of retinal blood veins network complexity description, ophthalmological diseases early determination and quantitative diagnostics in development scientific and practical importance has method as recommendation is being done.

CONCLUSION

In this study, we considered the problem of determining the fractal dimension of complex geometrical structures using the mass–radius method. By analyzing the scaling relationship between the accumulated "mass" M(R)M(R)M(R) and the observation radius RRR, we showed that the logarithmic dependence log⁡M(R)\log M(R)logM(R) versus log⁡R\log RlogR can be successfully used to estimate the fractal dimension as the slope of the best-fit line. This approach provides an intuitive and computationally simple tool for characterizing fractal geometry in both synthetic and real data sets.

The numerical experiments carried out on benchmark fractals (such as classical deterministic structures and stochastically generated patterns) demonstrated that the mass–radius method is capable of reproducing their known theoretical dimensions with good accuracy. We also observed that the method remains relatively robust under moderate levels of noise and finite-size effects, provided that the scaling range is chosen carefully and an appropriate regression procedure is applied. These results confirm that the mass–radius approach is competitive with more traditional dimension estimation techniques, such as box-counting, while often being easier to implement in practice.

An important outcome of our investigation is the identification of practical guidelines for applying the mass–radius method: the choice of center points, the selection of radius intervals, and the use of statistical averaging all have a significant influence on the stability and precision of the estimated fractal dimension. Taking these factors into account leads to more reliable estimates and reduces the sensitivity of the method to local irregularities and sampling artifacts.

The obtained results indicate that the mass–radius method can be effectively used in a wide range of applications where fractal structures naturally occur, including image analysis, porous media, biological textures, and computer-generated graphics. In future work, it would be interesting to extend the approach to anisotropic and multifractal systems, as well as to combine the mass–radius method with modern optimization and machine learning techniques in order to further improve the robustness and automation of fractal dimension estimation.

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