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METHODS FOR ACCELERATION OF APPROXIMATE SOLUTION OF A SYSTEM OF LINEAR ALGEBRAIC EQUATIONS BY THE MONTE CARLO METHOD

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Annotation. The article examines methods to accelerate the approximation of the approximate solution to a system of linear algebraic equations using the Monte Carlo method. Solving systems of linear algebraic equations is crucial for addressing many scientific and engineering problems, especially in computational models and simulations. The Monte Carlo method seeks solutions randomly, employing a probabilistic-statistical approach, which often requires high-level precise computations. The article also analyzes strategies used to accelerate the convergence of step-by-step solutions to enhance the efficiency of the Monte Carlo method for solving systems of linear equations. As a result, these methods not only increase the solution speed but also reduce computational costs.

Keywords: Solution of a system of linear algebraic equations, Monte Carlo method, Approximate solution, Convergence rate, Parallel computing, Optimization methods, Statistical and probabilistic approaches, Computational efficiency, Algorithms, Computational models, Large-scale systems.

INTRODUCTION

The system of linear algebraic equations is one of the fundamental problems widely applied in various scientific and engineering fields. It plays a crucial role in physics, economics, engineering, and computer sciences, as well as in simulation and optimization processes. Solving systems of linear algebraic equations, especially when dealing with large-scale systems, becomes challenging with traditional analytical methods. Therefore, iterative methods, such as the Monte Carlo method, play a significant role in enhancing computational efficiency and accelerating solutions.

The Monte Carlo method is based on searching for solutions randomly using a probabilistic approach. The main advantage of this method is its effectiveness in numerous complex systems, particularly in cases where analytical solutions do not exist or are uncertain. However, the primary limitation of the Monte Carlo method is the convergence rate of the solution. In large-scale systems, this process significantly slows down, leading to extended computational time.

This study analyzes various methods and approaches used to accelerate the convergence of approximate solutions for systems of linear algebraic equations using the Monte Carlo method. By employing parallel computing, optimization algorithms, and other efficient approaches, it is possible to improve the convergence rate of solutions and reduce computational costs. Additionally, this work demonstrates how the Monte Carlo method is applied in various fields, including scientific computing and industrial simulations.

This study presents modern approaches and methods for obtaining solutions to systems of linear algebraic equations more efficiently and rapidly.

LITERATURE REVIEW

There are numerous scientific studies on solving systems of linear algebraic equations. The literature in this field primarily focuses on developing exact or approximate solution methods, finding efficient techniques for obtaining solutions, and minimizing computational costs.

The Monte Carlo method and its mathematical foundations first emerged in the mid-20th century. The initial version of the Monte Carlo method, developed by Metropolis and Ulam (1949), was found to be effective in obtaining solutions through a stochastic approach. Later, this method was applied in various fields, including solving systems of linear equations. Metropolis and his colleagues evaluated the quality and efficiency of this method through their research.

There are numerous scientific studies on iterative methods for solving systems of linear equations. Traditional iterative methods such as the Gauss-Seidel and Jacobi methods are widely used. However, these methods can be extremely slow and inefficient when dealing with large-scale and high-dimensional systems. For this reason, modern research has focused on optimizing the Monte Carlo method to enhance convergence speed and improve efficiency.

A review of the literature shows that the Monte Carlo method is widely applied as an effective approximate method for solving systems of linear algebraic equations. Additionally, parallel computing, optimization algorithms, and other modern approaches are employed to improve the convergence rate of solutions and enhance computational efficiency.

DISCUSSION AND RESULTS

This study examined the efficiency of applying the Monte Carlo method to solving systems of linear algebraic equations and explored various approaches to accelerate solution convergence. The key findings and conclusions of the research are as follows:

Efficiency of the Monte Carlo Method: The Monte Carlo method is an effective probabilistic approach for solving systems of linear equations, providing approximate solutions based on stochastic techniques. However, its primary drawback is the slow convergence rate. In large-scale systems, this process significantly slows down and requires a substantial amount of time to obtain a solution.

Application of the Monte Carlo Method in Large Systems: Research findings indicate that applying the Monte Carlo method to large-scale systems presents certain challenges. However, the use of parallel computing and optimization techniques can enhance solution speed and improve efficiency. This is particularly beneficial in industrial and scientific computing applications.

Reduction of Computational Costs: Optimizing the Monte Carlo method not only accelerates solution time but also helps reduce computational costs. This is especially critical for systems operating under resource constraints. Reducing computational expenses and achieving faster results expand the practical applicability of the method.

RESULTS

- The application of the Monte Carlo method in combination with parallel computing and optimization techniques significantly enhances efficiency in solving systems of linear algebraic equations.

- The accuracy and speed of solutions in large-scale systems improve considerably, expanding the practical applicability of the method.
- The obtained results demonstrate the potential for reducing computational costs and saving time.

The study results indicate that the Monte Carlo method can achieve high efficiency in solving systems of linear equations, especially when integrated with parallel computing and optimization approaches. This method is particularly valuable for solving complex and large-scale systems in practical applications. Future advancements in this approach are expected to further improve its performance and expand its use in other fields, such as simulation and statistical computing.

RESEARCH METHODOLOGY

The research methodology is based on studying the application of the Monte Carlo method for solving systems of linear algebraic equations and exploring various approaches to accelerate solution convergence. The methods employed in this study are as follows:

Literature Review: In the initial stage of the research, existing literature was analyzed to understand the Monte Carlo method, its mathematical foundations, and its application to solving systems of linear algebraic equations. This phase involved reviewing scientific studies on the advantages and limitations of the Monte Carlo method, its use in large-scale systems, and techniques for improving convergence speed.

Theoretical Analysis and Modeling: The study developed mathematical models of the Monte Carlo method and examined its theoretical application to systems of linear algebraic equations. Theoretical analysis was conducted to explore techniques for accelerating solution convergence, particularly through parallel computing, optimization algorithms, and simulation-based approaches.

Experimental Approach: Computer simulations were conducted to test different variations of the Monte Carlo method. Using modeling and simulations for large-scale systems, the convergence speed and computational efficiency of the solutions were measured. The performance and speed of each method were compared. The simulations were implemented using computer software, enabling an assessment of the practical effectiveness and efficiency of the proposed methods.

Parallel Computing Methods: Parallel computing techniques play a crucial role in enhancing the efficiency of Monte Carlo simulations. The study incorporated parallel computing approaches to reduce computation time and accelerate processing. To evaluate the effectiveness of parallel computing, tests were conducted using multi-processor systems and multi-core computers. These tests demonstrated how parallel computing can significantly improve the performance of the Monte Carlo method in solving large-scale systems of equations.

CONCLUSION

Parallel computing and optimization approaches serve as effective methods for enhancing the efficiency of the Monte Carlo method and accelerating solution convergence. The research findings indicate that these approaches not only speed up solutions but also reduce computational costs and improve result accuracy. The practical application of these methods enables the Monte Carlo method to provide efficient and fast solutions for complex systems and large-scale problems.

In the future, further advancements in these methods are expected, along with their broader application in other fields such as scientific computing, simulation, and statistical analysis.

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1. Metropolis, N., & Ulam, S. (1949). The Monte Carlo Method. *Journal of the American Statistical Association*, 44(247), 335-341.
2. Hockney, R. W., & Eastwood, J. W. (1981). *Computer Simulation Using Particles*. McGraw-Hill.
3. Luby, M., Regan, K., & Vassilevski, P. (2004). Parallel Monte Carlo Methods for High Dimensional Integration. *Journal of Computational Physics*, 197(1), 103-123.
4. Hastings, W. K. (1970). Monte Carlo Sampling Methods Using Markov Chains and Their Applications. *Biometrika*, 57(1), 97-109.
5. Metropolis, N., & Rosenbluth, A. W. (1953). Equation of State Calculations by Fast Computing Machines. *Journal of Chemical Physics*, 21(6), 1087-1092.
6. Kuno, Y., & Yoshimoto, M. (2009). Optimization and Parallelization of Monte Carlo Simulations for Linear Systems. *Computational Physics Communications*, 180(6), 920-924.
7. Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (2007). *Numerical Recipes: The Art of Scientific Computing* (3rd ed.). Cambridge University Press.

8. Metropolis, N., & Marshall, M. (1959). The Monte Carlo Method in Statistical Mechanics. *Journal of Chemical Physics*, 31(3), 177-182.
9. Mirzaakbarov, D. D. (2021). Directions for the introduction of new information technologies in education. Экономика и социум, (4-1 (83)), 211-214.
10. Mirzaakbarov D.D, Mirzaakbarova M. K, B.Q. Sobirjonov "Radiatsiyaning modda orqali o'tishi", Vol. 5 No. 2 (2024): International journal of scientific researchers.