USING THE INVERSION ALGORITHM TO GENERATE 2D AND 3D DIMENSIONAL FRACTALS

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Abstract

The concept of circular inversion is introduced into the set of star shapes, and the algorithm for generating a circular inversion fractal uses a generalized substitution for the circular transformation. As a result, a star-shaped inversion fractal is obtained. The presented examples show that it is possible to obtain a wide variety of fractal patterns using the proposed method, and these patterns differ from those obtained by the circular inversion method. In addition, since a circle is a set of star shapes, the proposed generalization allows for a very easy and intuitive transformation of the circular inversion fractal.

Keywords: Inversion, fractal, IFS, Apollonia, star-shaped inversion fractals.

The concept of fractals was first introduced into science by Mandelbrot in the 1970s [1]. Building on Mandelbrot's conjectures, Barnsley put forward several innovative ideas that shed light on the practical aspects of fractals. He introduced a method for modeling natural fractals and used the concept of an iterative function system (IFS) as a generative tool. Fractals have since been used in many applications in pattern recognition [3], image processing [4], computer graphics [5], and even in medicine,[6] and archaeology[7]. Fractals can be used in geometric design[8], computer graphics [9], image processing [10] and even in medicine [11].

Circular inversion fractals. The inversion of circles (inversion of circles), whose Loci plane is mentioned in Frege's book Apollonius, has been widely used in geometry since Apollonius introduced the inversion of circles.

Definition 1. C - center Let *o be a circle with* radius *R* and let *p* be an arbitrary point outside *o* . If p' If $r(t) = o + t(p - o)$ is a point on the ray, where $t \in [0,\infty)$, then the following equation is satisfied:

$$d(o,p)\cdot d(o,p') = R^2, (1)$$

here is $d:\mathbf{R}^2 \times \mathbf{R}^2 \rightarrow [0,\infty)$ the Euclidean metric and p' The reciprocal of p for a circle C *(Figure 1). The point o is called* the inversion center, and C is the inversion circle. The transformation that takes p and p' turns it into is called the circular inversion transformation and is denoted by $I_{C.}$

(1) From the relation we can obtain the algebraic form of the circular inversion transformation. If we say $o = (x_o, y_o)$ and $p = (x_p, y_p)$, then $I_{C\ divides\ by}$ e into a formula of the form :

$$p' = I_C(p) = (x_0, y_0) + \frac{R^2}{(x_p - x_o)^2 + (y_p - y_o)^2}(x_p - x_o, y_p - y_o). \qquad (2)$$

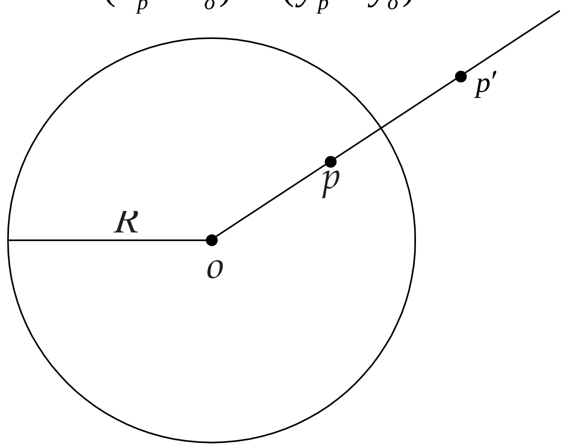

Figure 1: Inversion of p for a circle with center o and radius R.

By definition 1 , the point p is any point other than o , *but the definition* can also be applied to *o [10]. If $p = o$, then $I_C(o) = \infty$* and if $p = \infty$, then $I_C(\infty) = o$. As a result, I_C $\hat{\mathbf{R}}^2 = \mathbf{R}^2 \cup \{\infty\}$ is defined as.

Definition 2. Simple P A polygon is stellate if there is a point z that is not outside P , *and* a line segment for all points $\overline{zp}$ *p in P* lies entirely inside P. The above property is true. The location of the points z is the kernel of P.

All convex polygons are star polygons, which contain a core. The core of a star polygon is always convex. Figure 2 shows an example of a star polygon.

a star-shaped polygon can be generalized to a set as follows:

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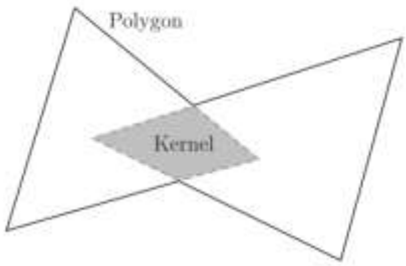
Definition 3. $\mathbf{R}^2$ of A set *S* is a star if for every $p \in S$ point $z \in$ *int* If there is a point *S* (where int denotes the interior of $\overline{zp}$ *SS*), *then the line segment* lies entirely inside *S*. The point *z* that is divided by e along its trajectory is the kernel of *S*.

Figure 2: Star-shaped polygons and their cores



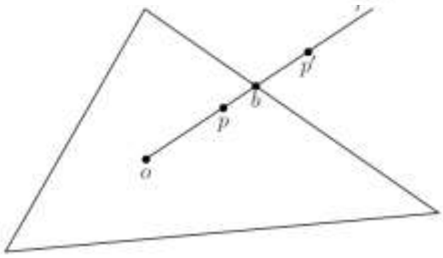


Figure 3: Inversion of *p* for a given set of star shapes (triangles) .

S is a set of stars and Let there be a point *o* belonging to the kernel of *S*. Furthermore, assuming that there are points *p* other than *o* , *the inversion of p* with respect to *S* is determined. For this, a *ray is passed* from *o* to *p* - *o*. That is, $r(t) = o + t(p - o)$ the boundary and intersection *b of S* for , $t \in [0, \infty)$, are determined (Fig. 3). Since *S* is a star and *o* belongs to its core, the point *b* is unique. If the following equation holds, it is called the inversion of *p* with respect to *S* :

$$d(o,p) \cdot d(o,p') = [d(o,b)]^2. \qquad (3)$$

For a circular inversion, the point *o* is called the center of inversion. *p* The transformation that takes and turns it into p' is called the inverse transformation of a star-shaped set and is denoted by I_S. Of course, in a similar way, the definition of I *S can be generalized to* *o* , so that $I_S(o) = \infty$ and $I_S(\infty) = o$. As can be seen from the above configuration, the inversion transformation of a star-shaped set is given by the inversion center *o and* can be different for the same set *S*.

I_S The algebraic expression for e is very similar to the following formula I_C. If $o = (x_0, y_0)$ and $p = (x_p, y_p)$ If , I *S is divided by* e as follows :

$$p' = I_S(p) = (x_o, y_o) + \frac{[d(o,b)]^2}{(x_p - x_o)^2 + (y_p - y_o)^2}(x_p - x_o, y_p - y_o). \quad (4)$$

S $\hat{\mathbf{R}}^2$ into three components: B = int S (the boundary component), $U = \hat{\mathbf{R}}^2 \setminus S$ (the non-boundary component), and ∂S. The star-shaped set inversion transformation has the following properties similar to the circle inversion:

1. I_S B and U are interchanged,
2. I_S is an identity in C,
3. I_S - contraction in U and Expansion in B,
4. I_S involution, that is, all $p \in \hat{\mathbf{R}}^2$ for $I_S(I_S(p)) = p$.

and properties of the star-shaped inversion set transformation, an algorithm for generating approximations of star-shaped inversion fractals is constructed (Algorithm 1). This algorithm is a modified version of the stochastic inversion algorithm proposed by Frame and Cogevina [10]. The modification consists in using the star-shaped inversion transform instead of the inversion circle transform.

Algorithm 1: Random inversion algorithm.

Login : $S_1, \ldots, S_k$ - star-shaped sets with selected inversion centers, p_0 - starting point outside $S_1, \ldots, S_k$, $n > 20$ - number of iterations.

Result: Limiting approximation of a finite set (a set of star- shaped inversion fractals).

$j = \{ 1, \ldots, k \}$ random number from

$p = I_{S_j}(p_0)$

for $I = 2$ **until** n **yes**

{

$l = \{1, \ldots, k\}$ is a random number from

while $j = l$ **or** $inSet\ (S_l, p)$ **yes**

$l = \{ 1, \ldots,$ random number from $k \}$

$j = l$

$p = I_{S_j}(p)$

if $I > 20$ **then**

Plot p

}

generate the fractal approximation, we need inversion transformations of the star set, a starting point outside the entire star set, and a series of iterations. First, an inversion is randomly selected and used to transform the starting point. Since the starting point is outside the star set defining the transformation , that is, it falls within the infinite U component of the transformation, and according to property 1, the transformed point is a transformation in the boundary component *B of the transformation.* Then, it randomly selects a transformation at each iteration with two constraints. The first constraint is that the points are not transformed by the transformation used in the previous iteration. This constraint follows from the involution of the I S involution (expression 4). Without this constraint, the number of points that can be approximated to the fractal can be reduced , *and* *more* iterations are required for better approximation. The second constraint is that the transformation of a point with a transformation that lies within the boundary component B cannot be performed. This limit is a consequence of Expression 3. The transformation is not an expansion , but a contraction, because allowing the transformation to be an expansion would push out the transformed points and complicate the approximation. The contraction also proves the convergence of the algorithm . If a change is selected, it will be converted to the points from the previous iteration. As in the random iterative algorithm for IFS , the first few points are skipped because they are not part of the approximation. The algorithm skips the first 20 steps.

When the algorithm reaches a limit through iteration, the generated points (orbits of the initial point) converge to a finite set of limits, i.e., the limiting set of point trajectories. If some orbit $p_{i\,is}$ in the set of stars S_j, then the next orbit is at point $p_{i+1}\, I_{Sj}(p_i)$ may be e.g. The term finite set of limits (for circles) was introduced by Clancy and Frame. The proof of the convergence of the algorithm is similar to the proof of the circle inversion algorithm presented by Smith .

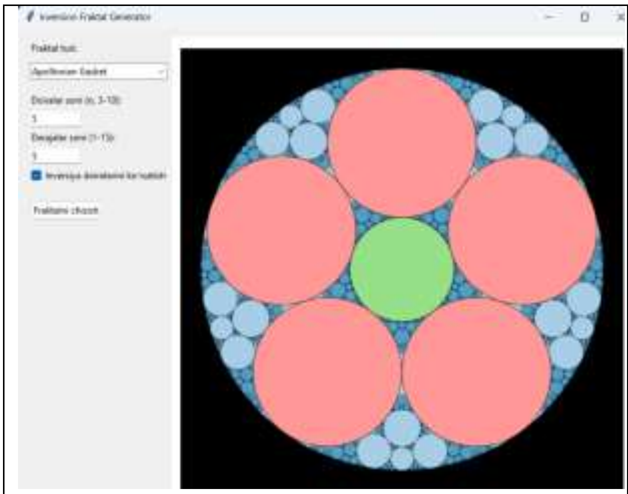
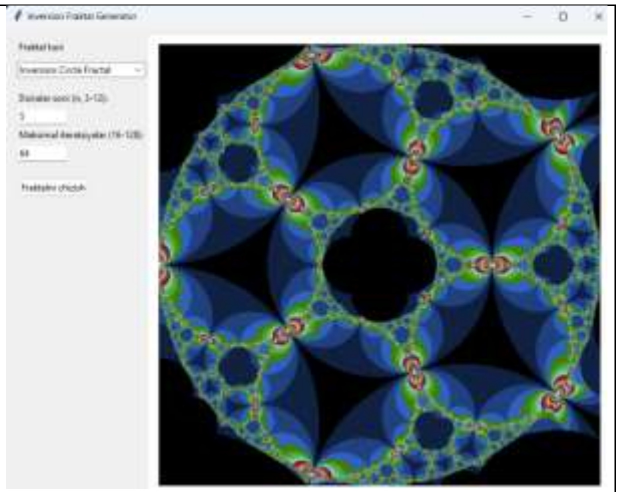
The fractal approximation obtained by the algorithm is located in the region enclosed by a set of star shapes that define the transformations applied in the algorithm. This follows from the second constraint and expression (1). The second constraint assumes that the point to be transformed is in the infinite component of the transformation , and expression (1) determines that the point after the transformation is in the boundary component. Each point generated in this way lies in the finite component of some transformation applied in the algorithm.

Naturally, a circle is a set of star shapes. If the center of the circle belonging to the nucleus is the inversion center , the inversion transformation of the star-shaped set reduces from Syeck to the circle inversion transformation. But for circles, any point can be taken from the nucleus inside. Examples show that changing the inversion center of circles allows you to deform the original circle inversion fractal [11].

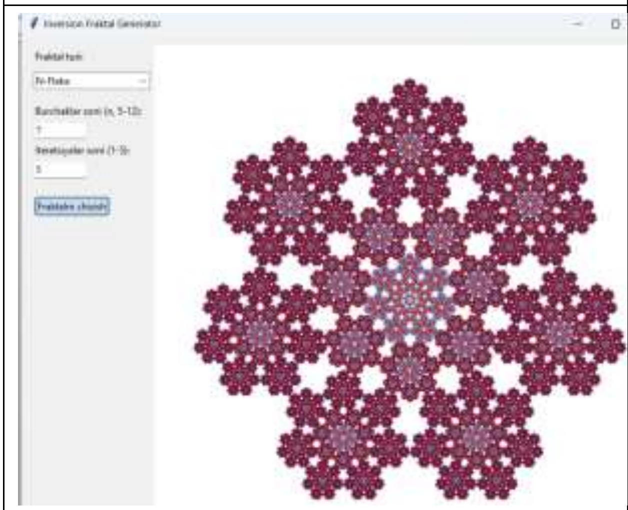
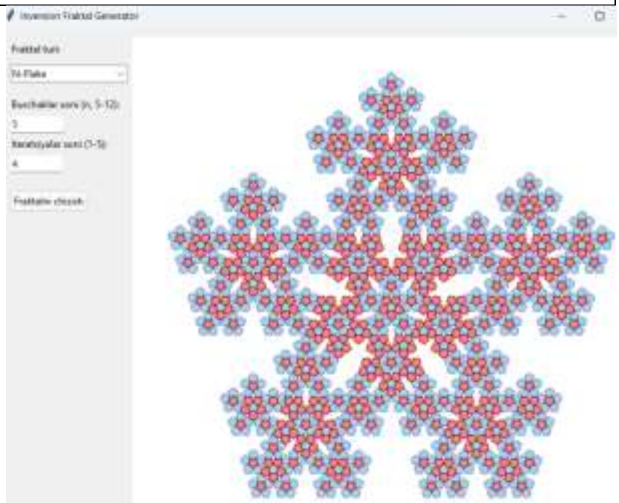
existing algorithm to find intersections with rays and determine whether a point is present in a given shape when performing a star set transformation . In addition, if polynomial curves are also passed when modeling the star-shaped boundary, there are corresponding algorithms to find intersections with rays and inclusion tests. In general, this task can be difficult to solve and therefore complicated to implement.

Although the study only shows *2D changes, the above ideas can be extended to* 3D . The set used in *3D* should have similar properties *to* the set of stars in *2D* .

The example in Figure 4 shows the result of changing the inversion center in a circular inversion fractal. The fractal is represented by eight circles. Figure 4(a) shows the fractal obtained by the Frame and Kogevin method . Figure 4(b) shows that the center of inversion of the large circle has been moved diagonally closer to the boundary of the circle . Note that the shape of the fractal changes and the points are expanded. Figure 4(c) shows that the center of inversion of the small circle has been moved closer to the boundary of the circle. Moreover, in this case, the shape follows the point. Figure 4(d) shows the change in the inversion centers of all the large circles and the inversion centers of the two smaller circles (the upper and lower positions in the figure, respectively). The center of the large circle is shifted asymmetrically . This caused the shape to lose its central symmetry, but the axisymmetricity was preserved. If the inversion center is completely moved asymmetrically , an asymmetric pattern is obtained. As can be seen from these examples , a circular inversion fractal can be easily deformed.

(a) (b)

(c) (d)

Figure 4 : An example of changing the inversion center along a circle in a set of circular and star-shaped inversion fractals .

As a result of this research, an algorithm was developed to generalize the circular inversion transformation to a set of star shapes and, as a result, to create a circular inversion fractal, and star-shaped inversion fractals.

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